Quantum Communication in Spin Systems With Long-Range Interactions

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Abstract

We calculate the fidelity of transmission of a single qubit between distant sites on semi-infinite and finite chains of spins coupled via the magnetic dipole interaction. We show that such systems often perform better than their Heisenberg nearest-neighbour coupled counterparts, and that fidelities closely approaching unity can be attained between the ends of finite chains without any special engineering of the system, although state transfer becomes slow in long chains. We discuss possible optimization methods, and find that, for any length, the best compromise between the quality and the speed of the communication is obtained in a nearly uniform chain of 4 spins.

1 Introduction

Quantum Information Processing (QIP) offers several advantages over classical computation in solving problems associated with large and/or dynamical systems. One of the main goals of current research in the field is to find ways and means of reliably transmitting quantum information, encoded in quantum bits (or *qubits*), over arbitrarily large distances. In order to ensure the information is received as the sender intended, the qubits must be protected from interacting with the environment in any way. For this reason, until now the qubits of choice have mostly been photons, which have an extremely small interaction cross section and can thus be used to generate quantum states that are sufficiently robust to perform protocols such as quantum cryptography and teleportation ([1, 2, 3, 4, 5, 6]).

However, in recent years much effort has been dedicated to studying systems in which quantum information is encoded in *stationary* qubits, and is propagated from one part of the system to another by the interaction between the system's components. One of the simplest geometries in which this can be achieved is a one-dimensional chain of interacting particles, where the qubit is encoded in some internal degree of freedom (which we call 'spin', using $|0\rangle$ for spin 'up' and $|1\rangle$ for spin 'down'). Originally an exchange-coupled chain of spins with a constant nearest-neighbour (nn) interaction was studied [7]; it was found that a qubit could be transferred with a fidelity exceeding the maximum classical value in a time that grows polynomially with the length of the chain. Subsequently, it was shown that such simple chains allow transmission fidelities arbitrarily close to unity also if the qubit is taken to be a carefully designed 'wave packet', provided the sending and receiving parties can access a sufficiently large portion of the chain [8]. Stronger results can be found for more complex systems: in the absence of structural imperfections, an XY Hamiltonian on a hyper-cubic lattice allows perfect state transfer [9, 10, 11], as does a pair of parallel spin chains [12], or a spin chain acting as a quantum wire connecting two qubits [13]. Plenio *et al.* [14] have studied the situation for chains of harmonic oscillators (i.e. where each particle on the lattice possesses a continuous, rather than a discrete, degree of freedom), while Hartmann *et al.* [15] have recently found that quantum information can be made to propagate with arbitrarily high fidelity through both oscillator and spin chains near a quantum phase transition, provided the ground state and the lowest excited state of the system are not degenerate. However, this transfer is exponentially slow; more rapid transmission is possible at the quantum critical point, but at some cost to the fidelity.

It has also been shown that high fidelities can be attained by engineering the strength and the nature of the interactions between the spins [16]. However, this requires structures that would be very difficult to manufacture, both because the communicating parties would need to have an extremely high degree of control over the system, and because the component spins would have to exhibit nn couplings only, with precisely defined strengths. This is clearly an idealization, because long-range interactions are also likely to be present. Previous work has been dedicated to systems more 'realistic' from this point of view. Kay *et al.* have studied finite spin chains in which the total Hamiltonian accounts for the presence of local magnetic fields and a potential of the form of the magnetic dipole interaction [17]. The approach adopted in this case was to pre-determine a spectrum of eigenvalues that would ensure perfect state transfer for all chain lengths, and subsequently derive the corresponding local fields and the inter-spin distances by solving an inverse eigenvalue problem. This method is applicable to a Hamiltonian

containing any number of parameters, and could provide very useful theoretical guidelines on the optimal way to structure a system, although, once again, the gap between theory and experiment may prove difficult to bridge.

In this work, by contrast, we investigate simple one-dimensional arrays of spins interacting via a pure magnetic dipole interaction. We allow no site-specific locally-tunable fields; nevertheless, we show that fidelities for quantum state transfer closely approaching unity can be attained between the ends of finite chains, without any special engineering of the system. Furthermore, because of the long range interaction, the transfer rate grows polynomially in the system size, rather than exponentially, as in the case studied by Hartmann *et al.* [15]. Our results may be relevant to two-level atoms in atomic traps [18, 19] or to one-dimensional arrays of endohedral fullerene species encapsulated within carbon nanotubes [20], as well as to natural magnetic dipolar systems such as $LiHoF_4$ [21], and finite spin chains of a more complex design, for example engineered from arrays of quantum dots [22].

2 The System

We build on the work done by Bose [7] on transferring quantum information through an infinite, uniform chain exhibiting isotropic nn interactions only. In this system, the qubit is represented by a single flipped spin, which propagates between different sites in a manner defined by a time-independent Hamiltonian of the form:

$$H = -\frac{J}{2}\delta_{i+1,j}\sum_{\langle i,j\rangle}\sigma^i\cdot\sigma^j - B\sum_{i=1}^{N}\sigma_z^i \tag{1}$$

where N is the number of spins in the chain, $\sigma^{\mathbf{i}} = (\sigma_x^i, \sigma_y^i, \sigma_z^i,)$ are the Pauli spin matrices for the i^{th} spin, $B > 0$ is a uniform magnetic field, and $(J/2)\delta_{i+1,j}$ is the coupling strength between spins i and j, which is non-zero for nearest-neighbouring spins only. If the ground state of the system is expressed as $|\downarrow\rangle\bigotimes|\downarrow\rangle\bigotimes...\bigotimes|\downarrow\rangle$, it has been shown that, provided the chain is sufficiently short, perfect or near perfect state transfer can be achieved by simply letting an initial state of the form $|\uparrow\rangle\bigotimes|\downarrow\rangle\bigotimes...\bigotimes|\downarrow\rangle$ evolve naturally in time according to the effects of $\mathbf{H}$. It is important to note that $[H, \sum_{i=0}^{N}\sigma_z^i] = 0$, that is, the Hamiltonian conserves the total magnetization $\mathbf{M}$ of the system, allowing the chosen initial state to evolve only into states in which *one* spin is flipped at any given time.

We propose to investigate the quality and efficiency of quantum state transfer through infinite and finite chains of spin-$\frac{1}{2}$ fermions coupled by long-range interactions having the form of the magnetic dipole interaction. These

two systems differ in that the finite chain has end points, whereas the infinite chain does not; indeed, due to the periodic nature of the infinite chain we will hereafter refer to it as a *ring*. We examine a simplified system in which any external magnetic field is constant and parallel to the axis joining the dipoles, which is chosen to coincide with the z direction. The alignment of the magnetic and dipole axes is not a trivial point, but a necessary condition to ensure that the total magnetization $\mathbf{M}$ remains a good quantum number, and allows us to work in the sub-space where only one spin is flipped with respect to the ground state, reducing the Hamiltonian from a $2^N \times 2^N$ to a $N \times N$ matrix. Within this subspace the effect of the magnetic field is to add a constant to the energies. We will hereafter omit this constant. Following the notation used in [7], we denote with $|0000...0\rangle$ the (unique) ground state of the system (i.e. all spins facing down, parallel to the external field) and with $|j\rangle$ the block of states in which the spin at the j^{th} site has been flipped from 0 to 1. For simplicity, we assume there are no thermally excited spin-flips in the system. We adopt a Hamiltonian of the form:

$$H_d = \frac{C}{r^3}[\mathbf{S_i} \cdot \mathbf{S_j} - 3\mathbf{S_i^z}\mathbf{S_j^z}] \tag{2}$$

where C is a constant, $\mathbf{S_i}$ and $\mathbf{S_j}$ are the total spin operators at sites i and j, and $\mathbf{S_i^z}$ and $\mathbf{S_j^z}$ are the respective z components. The value of C is determined by the type of particle in the chain. For a system of spin-$\frac{1}{2}$ fermions (e.g. electrons) C is given by:

$$C = \frac{\mu_0(\mu_B g)^2}{4\pi\hbar^2} \tag{3}$$

where μ_0 is the permeability of free space, μ_B is the Bohr magneton, g is the electronic Landè g-factor and $\hbar = h/2\pi$. Throughout this paper we will assume that $\mu_0 = \mu_B = \hbar = 1$, so that:

$$C = \frac{g^2}{4\pi} \tag{4}$$

We define a to be the spacing between neighbouring fermions. In this case, the strength of the interaction between nearest neighbours is:

$$\langle i|H_d|i \pm 1\rangle = \frac{C}{2a^3} \tag{5}$$

For the results shown we define our length, energy and time units by setting the nearest-neighbour separation and the interaction energy between nearest neighbours to unity. However, eqn.(5) implies the Hamiltonian has an overall scaling factor of $1/a^3$, so a uniform compression or expansion of the system

should only have the *quantitative* effect of re-scaling the system's energy by a constant. Therefore, provided the chain remains uniform and the number of component spins is fixed, the energy and performance of a chain of any size can be extrapolated by simply adjusting the value of a as necessary.

2.1 Rings

We initially consider a ring of N spins in its ground state. Our aim is to calculate the maximum fidelity of transmission of a qubit from site r to a distant site s, as a function of time and number of spins in the ring. We denote the initial and final states of the system by $|r\rangle$ and $|s\rangle$ respectively. The expression for the maximum fidelity of quantum state transfer is given by [7] as:

$$F_{r,s}^{N}(t) = \frac{\left|f_{r,s}^{N}(t)\right|}{3} + \frac{\left|f_{r,s}^{N}(t)\right|^{2}}{6} + \frac{1}{2} \tag{6}$$

where $f_{r,s}^{N}(t)$ is the propagator, which is calculated from the following:

$$f_{r,s}^{N}(t) = \sum_{m=1}^{N} \langle r|m\rangle\langle m|s\rangle e^{-iE_m t} \tag{7}$$

We assume that the eigenvectors $|m\rangle$ of the system can be expanded using the basis formed by the $|j\rangle$ states. Imposing periodic boundary conditions allows us to express these eigenstates as Bloch states, so that for a ring of circumference $L = Na$:

$$|m\rangle = \frac{1}{\sqrt{N}} \sum_{j=0}^{N-1} e^{ik_m ja}|j\rangle \tag{8}$$

The propagator can then be re-expressed as:

$$f_{r,s}^{N}(t) = \frac{1}{N}\sum_{m=1}^{N} e^{ik_m a(r-s)}e^{-iE_m t} = \frac{1}{N}\sum_{m=0}^{N-1} e^{i\frac{2\pi m}{N}(r-s)}e^{-iE_m t} \tag{9}$$

Using eqn. (8), we calculate the energies of the system, which can be written as:

$$E_m = \langle m|H_d|m\rangle = \frac{1}{N}\sum_{i,j}^{N-1} e^{ik_m a(j-i)}\langle i|H_d|j\rangle \tag{10}$$

We require only matrix elements for which $i \neq j$, since by symmetry the diagonal terms of H_d are independent of N, and therefore change the energy

of the system by a constant shift. For evenly spaced spins with nearest-neighbour separation a, these off-diagonal terms are:

$$\langle i|H_d|j\rangle = \frac{C}{2|r_j - r_i|^3} = \frac{C}{2a^3|j-i|^3} \tag{11}$$

As eqn. (11) only depends on the difference $|j-i|$, we can reduce the double summation to a single sum over j by fixing a value of i. For convenience we choose $i = 0$, so that:

$$E_m = \frac{C}{2a^3} \sum_{j=-\frac{N}{2}}^{\frac{N}{2}} e^{ik_m ja} \frac{1}{|j^3|} = \frac{C}{a^3} \sum_{j=1}^{\frac{N}{2}} \frac{\cos(k_m ja)}{j^3}. \tag{12}$$

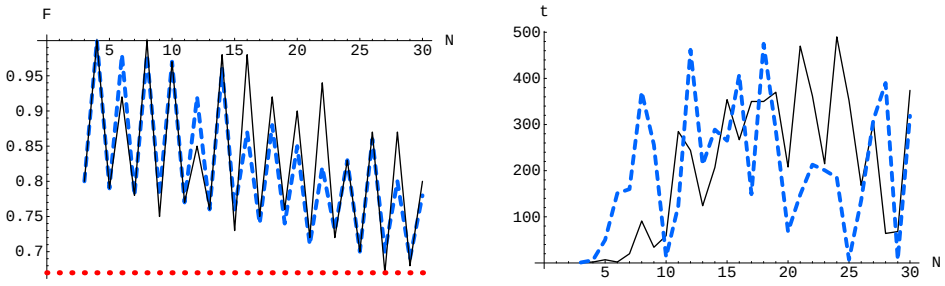

Figure 1: (Colour online) Figure (a) shows the maximum fidelity that can be achieved in transferring an input state $|1\rangle$ to an output state $|\frac{N}{2}+1\rangle$ or $|\frac{N+1}{2}\rangle$ on a ring of spins coupled by dipole-dipole (dashed blue curve) or nearest-neighbour (black curve) interactions, as a function of N. The red dotted line at $F = 2/3$ indicates the highest fidelity for classical transmission of a quantum state. Figure (b) shows the time at which the fidelity first peaks in this system. Units are as specified in Section 2.

Fig. 1(a) shows the maximum fidelity of state transfer for rings of $N = 3$ to $N = 30$ spins, when the sending and receiving parties are located at diametrically opposite sites, for which $|r\rangle = |1\rangle$ and $|s\rangle = |\frac{N}{2} + 1\rangle$ or $|s\rangle = |\frac{N+1}{2}\rangle$ for even and odd N respectively. We have assumed that transfer occurs along the arc joining sites r and s, though this may not be exactly the case. Fig. 1(a) also shows the performance of a ring in which the spins are coupled by nn interactions only. We note that for $N > 3$ the performance of the nn-coupled ring is slightly better unless $N = 6$ or $N = 12$. We also find that the times of optimum transfer tend to rise as we increase N in both the dipole and the nn-coupled rings.

2.2 Single Qubit Transfer in Uniform Chains

We now extend the previous analysis to a finite chain, calculating the full Hamiltonian of the system, which has the form:

$$\langle i|H_d|j\rangle = \frac{C}{2a^3|j-i|^3} \tag{13}$$

$$\langle j|H_d|j\rangle = -\frac{C}{2a^3}\sum_{\langle k,l\rangle}\frac{1}{|k-l|^3} + \frac{C}{a^3}\sum_{i\neq j}\frac{1}{|j-i|^3} \tag{14}$$

where $\langle 000..0|H_d|000..0\rangle = -\frac{C}{2a^3}\sum_{\langle k,l\rangle}\frac{1}{|k-l|^3}$ is the ground state energy of the system.

The fidelity of state transfer between the ends of the chain is obtained by taking $|r\rangle = |1\rangle$ and $|s\rangle = |N\rangle$. We find that $F^N_{1,N}(t)$ exhibits three 'trademark' features.

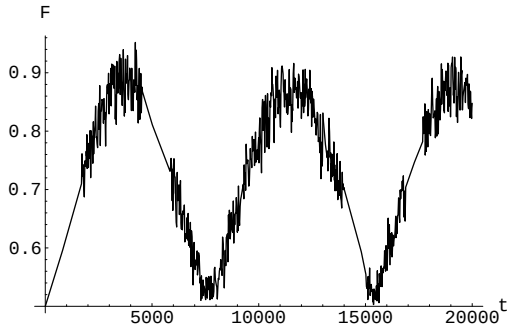

Figure 2: The evolution in time (abscissa) of the fidelity of state transfer between sites 1 and 10 of a uniform chain of magnetic dipole-coupled spins. We note the regularity of the oscillation and the high value of $F(max)$. Units are as specified in Section 2.

First of all, the maximum value of $F(t)^N_{1,N}$ is close to unity. Secondly, the value of $F^N_{1,N}(t)$ oscillates between 1/2 and the maximum (which we call F_{max}) with a regular frequency, which is generally quite small, implying that state transfer occurs slowly (fig. 2). Finally, the period of oscillation of $F^N_{1,N}(t)$, which we call T, is uniquely defined by the energy splitting $\Delta\lambda$ between the two lowest eigenvalues of $H_d(N)$. The transfer process is therefore dominated by the beating of two nearly degenerate states localized near the ends of the chain. This behaviour is explained by the variation of the on-site energies of the spins as a function of j, shown in fig. 3; it is immediately evident that the most favourable positions for a spin to flip are sites 1 and N. Consequently, states $|1\rangle$ and $|N\rangle$ are the most strongly coupled to the system's (two) bound states, which are shown in fig. 4. In this system, this phenomenon is a natural

consequence of the geometry, but systems in which the spin flip energy is specifically chosen site by site have also been studied [23].

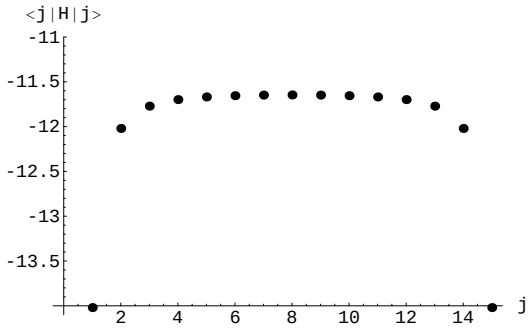

Figure 3: The on-site energy as a function of the site j of the spin flip for a chain of 15 magnetic dipole-coupled spins. We note that the energies at sites 1 and 15 are much lower than the rest. Units are as specified in Section 2.

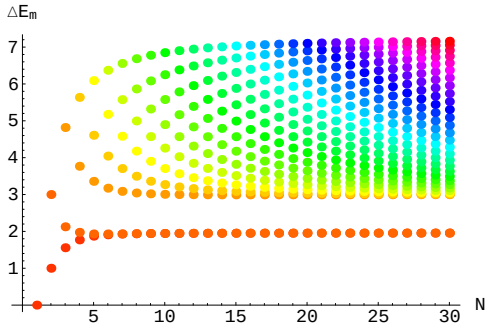

Figure 4: (Colour online) The energy splitting ΔE_m between the ground state $|0000..0\rangle$ and states with a single flipped spin for a uniform chain of N spins, showing the evolution of the two bound states. Values of ΔE_m of the same index m, counting from the bottom of the spectrum, are shown in the same colour. Units are as specified in Section 2.

The period of $F^N_{1,N}(t)$ is related to $\Delta\lambda$ by:

$$T = \frac{2\pi}{\Delta\lambda} \tag{15}$$

Consequently, the time at which $F^N_{1,N}(t)$ first peaks is:

$$t(F_{max}) = \frac{T}{2} = \frac{\pi}{\Delta\lambda} \tag{16}$$

This time rises with chain length, as the splitting $\Delta\lambda$ decreases with increasing N.

A summary of our results for $N = 2$ to $N = 23$ spins is shown in Fig. 5(a). We note that, in addition to $N = 2$, $N = 3$ and $N = 4$ also give perfect transfer, and in general $F_{max} \geq 0.9$. This is a marked improvement on the performance of a nn-coupled chain (also shown in Fig. 5(a)); in particular, it seems that by replacing the nn couplings with dipole couplings we no longer obtain poor transfer when N is a multiple of 3 [7]. Unfortunately, we again observe a 'trade-off' between fidelity and time, which becomes particularly evident for $N > 6$. In fact, we find that at large N, $t(F_{max})$ goes as the cube of the chain length (fig. 5(b)). It is therefore evident that in long chains it will take an impractical length of time to complete the protocol unless the system can be optimized in some way.

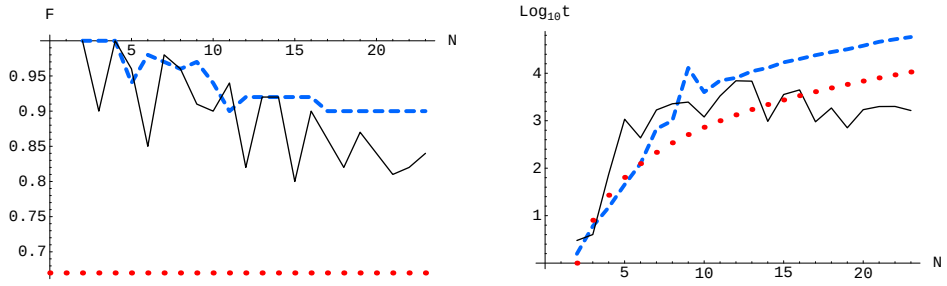

Figure 5: (Colour online) Figure (a) shows the maximum fidelity that can be achieved in transferring an input state $|1\rangle$ to an output state $|N\rangle$ in a chain of spins coupled by dipole-dipole (dashed blue curve) or nearest-neighbour (black curve) interactions, as a function of N. The red dotted line at $F = 2/3$ indicates the highest fidelity for classical transmission of a quantum state. We note that the dipole-coupled chain almost always performs better. Figure (b) shows the time at which the fidelity first peaks in these two systems, plotted on a Log_{10} scale. The red dotted curve is the function $y = L^3$. We see that at large L the dashed blue and red dotted curves are parallel, indicating that the transfer time scales as the cube of the chain length. Units are as specified in Section 2.

2.2.1 Structural Optimization

We now analyze the efficiency of state transfer for a *fixed* chain length, as a function of the number of spins in the chain. We define τ as the transmission time giving maximum fidelity at unit chain length, i.e.:

$$\tau = \frac{t(F_{max})}{L^3} \tag{17}$$

Fig. 6 shows a plot of τ as a function of N, which reveals two interesting features. The first is the presence of a minimum at $N = 4$, which we will

discuss subsequently. The second is the fact that τ tends to a constant at large L. This indicates that, above a certain threshold value of N, the evolution of the system is determined almost exclusively by the magnetic dipole coupling between spins 1 to q with spins $N-q$ to N, irrespective of the number of spins that separate these two 'clumps'. To explore this

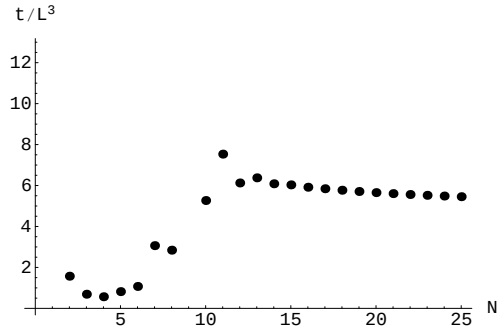

Figure 6: The behaviour of τ as a function of the number of spins in the chain. Note the minimum at $N = 4$ and the flatness of the curve for $N > 15$. Units are as specified in Section 2.

hypothesis, and determine the behaviour of τ for large N, we work with states $|B\rangle$ and $|E\rangle$ localized at the beginning and end of the chain, which are the bound-state eigenfunctions of a semi-infinite chain extending to the right and the left, respectively. We take:

$$|B\rangle = \sum_{n=1}^{q} a_n |n\rangle \tag{18}$$

and

$$|E\rangle = \sum_{n=1}^{q} a_n |N+1-n\rangle \tag{19}$$

The energy splitting of the two lowest eigenvalues of $H_d(N)$ in a finite chain is:

$$\Delta\lambda = 2\langle B|H_d|E\rangle \tag{20}$$

From (13):

$$\langle i|H_d|j\rangle = H_d(|i-j|) \tag{21}$$

Hence:

$$\langle B|H_d|E\rangle = \sum_{n,m=1}^{q} a_n^* a_m \langle n|H_d|N+1-m\rangle = \sum_{n,m=1}^{q} a_n^* a_m H_d(|N+1-m-n|) \tag{22}$$

We adopt a 'dummy' variable $X = |i - j|$, so that:

$$H_d(X) = \frac{C}{2a^3X^3} \tag{23}$$

$$\frac{\partial H_d(X)}{\partial X} = -\frac{3C}{2a^3X^4} \tag{24}$$

Using (22) and the fact that $L = a(N-1)$, we can expand $H_d(|N+1-m-n|)$ as a Taylor series to first order in $\delta = m + n - 2$. Then:

$$H_d(|L - \delta|) = H_d(L) - \delta \left.\frac{\partial H_d}{\partial X}\right|_{X=L} = \frac{C}{2L^3} + \frac{3Ca(m+n-2)}{2L^4} \tag{25}$$

Hence:

$$\langle B|H_d|E\rangle = \frac{C}{2}\left[\frac{1}{L^3}\sum_{n,m=1}^{q} a_n^* a_m + \frac{a}{L^4}\sum_{n,m=1}^{q} 3a_n^* a_m(m+n-2)\right] \tag{26}$$

$$= \frac{C}{2}\left[\frac{Q}{L^3} + \frac{aR}{L^4}\right] \tag{27}$$

with:

$$Q = \sum_{n,m=1}^{q} a_n^* a_m \tag{28}$$

$$R = \sum_{n,m=1}^{q} 3a_n^* a_m(m+n-2) \tag{29}$$

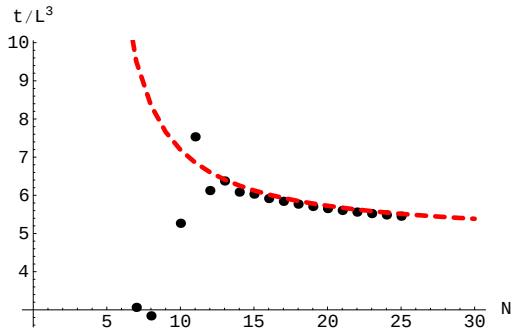

Figure 7: (Colour Online) Comparison between the predictions of our q-spin model (red dashed points) and the data calculated from the treatment of the system in its entirety. We note the model becomes increasingly accurate at large N.

We find it is possible to model the asymptotic behaviour of the system very accurately if we assign to the coefficients a_n the amplitudes of the ground

state eigenvector of the $q \times q$ sub-matrix generated by truncating the full Hamiltonian $H_d(N)$ for an arbitrarily large N (fig. 7). The values of a_n used to obtain the fit in fig. 7 come from the ground state eigenvector of the 4×4 sub-matrix of $H_d(14)$, and give $Q \approx 0.325$ and $R \approx -0.957$. These quantities show only a very weak dependence on N, so we have assumed them to be constants. The fact that $Q < 1$ shows that the transfer rate for chains of *many* spins is always less than that attained between two completely isolated spins; equation (28) indicates this is a result of interference between positive and negative components in the localized states $|B\rangle$ and $|E\rangle$.

Therefore, only chains with *few* spins can improve on the performance of a simple dipole pair, as shown by the minimum in the function $\tau(N)$ at $N = 4$ (fig. 6). This result, together with fig. 5(a), shows that, in a *uniform* chain, the best compromise between the quality and the speed of the communication is obtained with 4 spins. This occurs because for short chains the bound states at the two ends have a large overlap, i.e. there exist terms in eqn. (22) which simultaneously have significant positive values of $a_n^* a_m$ and small values of $|N+1-m-n|$. We have attempted to optimize the uniform 4-spin chain still further, and find it is possible to improve its performance slightly by modifying the positions of the inner spins while maintaining mirror symmetry. For a chain of unit length, this corresponds to taking $r_{1,2} = r_{3,4} \approx 0.314$ and $r_{2,3} \approx 0.373$, which yield a value $\tau \approx 0.512$. However, a comparison with $\tau \approx 0.568$ for a uniform chain ($r_{1,2} = r_{2,3} = r_{3,4} \approx 0.333$) shows the improvement is minimal.

2.2.2 Input and Output Optimization

We now investigate the effects of altering the initial and final states $|r\rangle$ and $|s\rangle$, while leaving the structure of the chain intact.
If the starting and ending points are chosen at random, the characteristic oscillation of $F_{r,s}^N(t)$ is lost, unless either $|r\rangle = |2\rangle$ and $|s\rangle = |N\rangle$, or $|r\rangle = |1\rangle$ and $|s\rangle = |N-1\rangle$. However, in both cases the signal is considerably noisier, and the maximum fidelity is greatly reduced. This is a result of the lesser efficiency of coupling to the bound states as one moves away from the ends of the chain (cfr. fig. 4).

Conversely, it is possible to boost the maximum fidelity to unity and smooth out all noise in the signal by encoding the states $|r\rangle$ and $|s\rangle$ in two or more adjacent spins[1](fig. 8). This is equivalent to adopting $|r\rangle = |B\rangle$ and $|s\rangle = |E\rangle$, where the a_n are now obtained from the first and last n coefficients of the ground state eigenvector of $H_d(N)$, with the additional condition that

[1]The possibility of improving state transfer by encoding a state in more than a single spin is also discussed in [8].

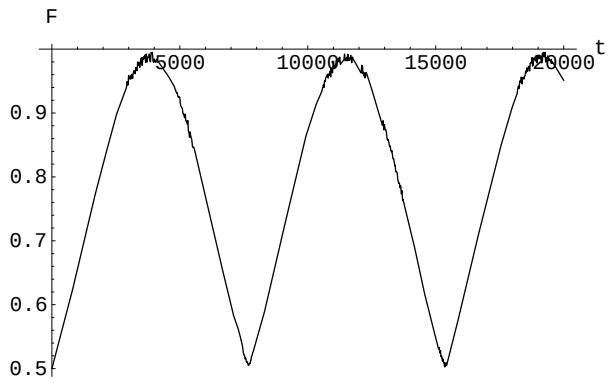

Figure 8: The evolution in time (abscissa) of the fidelity of transmission of an input state of the form $C_1|1\rangle + C_2|2\rangle$ to an output state of the form $C_9|9\rangle + C_{10}|10\rangle$ in a uniform chain of 10 dipole-dipole coupled spins. Comparing with fig. 2, note that the curve is smoother and the maximum fidelity has increased. Units are as specified in Section 2.

$\sum_n |a_n|^2 = 1$. This choice of input and output states leaves the transfer time unaffected.

3 Discussion and Conclusions

We present a scheme for transferring quantum information through infinite and finite chains of spins coupled via a pure magnetic dipole interaction. This differs from much previous work in that the dipole interaction is long-range, making for a system in which every spin interacts with all other spins in the system, rather than with nearest neighbours only. We find that, in general, the maximum fidelity achievable by using a dipole-coupled system to transfer a state between two maximally distant sites is greater or equal to that which can be attained in a system exhibiting nearest-neighbour interactions only. The *finite* chain, in particular, can be engineered to give unit fidelity by simply adjusting the placement of the spins and the input and output states. We have verified this result only for $L = 2$ to $L = 23$, but believe it extends to longer chains also.

The main weakness of both our systems is length of time taken to complete the protocol, which increases polynomially in the size of the system. However, we find that for a finite chain this obstacle can be considerably lessened by simply modifying the relative placement of the spins. Therefore, it does not necessarily preclude the possibility of being able to transmit information over longer distances on useful timescales. Furthermore, the protocol seems to be reasonably robust against errors in spin placement; if we define

a "failure rate" as the probability that the fidelity at time $t(F_{max})$ will fall below the classical value, we find that, in a uniform 4-spin chain of arbitrary length, a random error of 2 % on the placement of each spin yields a failure rate of approximately 5 %. The corresponding uncertainty on $t(F_{max})$ is significantly greater, but as $F^N_{1,N}(t)$ is a slowly varying function of time, it is quite unlikely that the fidelity at $t(F_{max}) \pm \delta t(F_{max})$ will have fallen significantly below the maximum. Therefore, the simple and predictable behaviour of the fidelity in time in a finite chain greatly increases the probability of carrying out successful state transfer. The long range interactions also open up the realistic possibility of measurements on individual spins, and it would be interesting to investigate in the future whether this can increase the speed of quantum state transfer as in Ref. [24].

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